SEQUENTIAL COMMUNICATION WITH EX POST PARTICIPATION

CONSTRAINTS

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Abstract

This paper examines an informed principal-agent game with ex post participation constraints for the agent. It shows that the players do not loss by communicating in turn among themselves rather than simultaneously if and only if the principal communicates first. It then considers any Bayesian allocation rule implemented in this bilateral asymmetric information framework. It provides necessary and sufficient conditions for sequential communication to be as efficient than simultaneous communication in implementing Bayesian allocation rules when the player with unbounded payoffs moves first.

Key words: asymmetric information, principal-agent, implementation, contract theory.

JEL codes: D23, D82.

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1 Introduction

Consider two agents. They must select a decision through a mechanism. Preferences are private information. Side-payments are allowed. One of the two agents has limited liability. Which mechanism should they use? According to the Revelation Principal, no mechanism Pareto dominates (in terms of ex ante payoffs) a direct revelation mechanism. It prescribes that the agents send simultaneously a message (about preferences) to an arbitrator who then selects a decision (with a side-payment). This paper investigates whether simultaneous communication can be replaced by sequential communication (in which agents communicate in turn among themselves) in direct revelation mechanisms and how such a sequential mechanism look like.

The agents might be the members of an organization who decide on a project size. Or two firms, a producer and a retailer, that contract on production levels and prices. In those examples, the agents are free to quit the organization, or to exit the contract, anytime (e.g. by going bankrupt). They are protected by limited liability. Payoffs must therefore be non-negative to insure participation. Furthermore, by defining who decides first on what, the hierarchical structure in organizations, or the assignment of decision rights in contracts, specifies a particular sequence of moves. The paper rationalizes the use of a sequential decision process. It also says something about its design.

With simultaneous communication, the allocations implemented (decisions and side-payments) must be Bayesian incentive compatible (BIC) for the two agents. In contrast, with sequential communication, they must satisfy the stronger dominant strategy incentive compatible (DSIC) condition for the agent who communicates last. The paper provides necessary and sufficient conditions for which the DSIC condition for the agent who has limited liability can be obtained for free (in terms of expected payoffs). Since, as argued by Crémer and Riordan [1], the DSIC condition has simpler informational and computational requirements, a sequential mechanism

\[\text{See Sappington [8] for a discussion on the limited liability assumption in the principal-agent model.}\]
is particularly adapted to deal with less informed or rationally bounded agents.

The first part of the paper focuses on the allocations implemented if one player, the “principal”, makes a take-it-or-leave-it contract offer to the other player, the “agent”, whose outside option is nil. It shows that simultaneous communication can be replaced by sequential communication if and only if the principal communicates first. This result holds as well if the principal has also bounded ex post payoffs.

The second part considers any allocations that are BIC for both partners and satisfy the ex post participation constraints for one. It provides necessary and sufficient conditions for which the stronger DSIC condition for the ex post participation constrained partner can be obtained for free. It therefore yields sufficient conditions for the use of sequential mechanisms in place of simultaneous mechanisms while leaving the two partner’s expected utility unchanged.

The first part builds on Maskin and Tirole [5]. They show that the principal does not loss by revealing her type before the agent, thereby imposing the stronger DSIC condition to the agent. Here I put the argument one step further by showing that the sequence of communication matters if the agent has limited liability: the principal can communicate first but not last. Put differently, imposing the DSIC condition in place of the BIC condition to the principal reduces her ex ante payoffs.

The second part is related to literature on the implementation of Bayesian mechanisms with quasi-linear preferences. Mookherjee and Reichelstein [7] show that, under sufficient conditions, the BIC constraints can be replaced by the DSIC constraints for all agents while leaving unchanged every agent’s expected payoff. Yet it requires to modify side-payments which affects payoffs. The present paper shows that, on the one hand, some agent’s payoffs might be negative. The DSIC condition

\footnote{These allocation are not necessarily the ones that emerge from an ex ante negotiation with extreme bargaining powers.}

\footnote{To be precise, Maskin and Tirole show that, in their adverse selection model but with quasi-linear preferences, the informed principal does not loose by revealing her type to the agent at the contracting stage, thereby moving incentive compatibility constraints from the interim to the ex post stage (Proposition 11 in the paper).}
might therefore not be compatible with ex post participation constraints.\(^4\) On the
other hand, with only two players, one being ex post participation constrained, BIC
for all can be replaced by BIC for one agent and DSIC for the other.\(^5\)

Crémer and Riordan \([1]\) show that, under sufficient conditions, BIC for all can
be replaced by BIC for one agent and DSIC for the others. However, they do not
address the participation, or individual rationality, issue at all.\(^6\) Here, I show that (i)
this result might hold as well with two agents when participation is an issue ex post
for one agent, (ii) who is DSIC constrained and who is BIC constrained matters.

2 Model and definitions

There two agents, hereafter called “players”, a principal \(P\) (“she”) and an agent
\(A\) (“he”). Each player \(i = P, A\) has preferences \(u_i(y, \theta_i)\) over a common decision
\(y \in Y \subseteq \mathbb{R}\) and private information \(\theta_i \in [\underline{\theta}_i, \bar{\theta}_i] \equiv \Theta_i\), with \(\underline{\theta}_i < \bar{\theta}_i\). \(\theta_i\) is referred
as \(i\)’s type. It is common-knowledge that \(\theta_P\) and \(\theta_A\) are independently distributed
according to the density \(f_i(\theta_i) > 0\) and cumulative \(F_i(\theta_i)\) for \(i = P, A\). The function
\(u_i\) is assumed thrice continuously differentiable, concave in \(y\), and increasing in \(\theta_i\). I
make the assumption of increasing marginal utility with \(\theta_i\) for every \(i = P, A\), known
as the “single-crossing property”.\(^7\)

Assumption 1 \(\frac{\partial^2 u_i}{\partial y \partial \theta_i} > 0\).

Players perform transfers among themselves. Denote \(t\) the net transfer (possibly
negative) from the principal to the agent. An allocation is a vector \((y, t)\). \(P\) and \(A\)’s

\(^4\)Proposition 1 shows that it is indeed the case for the allocation implemented in the informed
principal-agent game.

\(^5\)Proposition 2 provides conditions for which BIC for all can be replaced by BIC for one partner
and DSIC for the other.

\(^6\)In particular, they focus on the Bayesian implementation of the first-best levels of public good
which might not be compatible with the agents’ participation condition (see, e.g., Example 23.E.1.
in Mas-Colell et al. \([4]\)).

\(^7\)This condition ensures that a solution derived by recognizing only the “local” incentive con-
straints will also be globally incentive compatible (see the proof of Lemmata 1 and 2).
payoffs with the allocation \((y, t)\) in state \((\theta_P, \theta_A) \in \Theta\) are, respectively:

\[
U_P(y, t, \theta_P) = u_P(y, \theta_P) - t,
\]

and,

\[
U_A(y, t, \theta_A) = u_A(y, \theta_A) + t.
\]

A state of nature is a vector \(\theta = (\theta_P, \theta_A) \in \Theta \equiv \Theta_P \times \Theta_A\). The (total) surplus in state \(\theta\) is:

\[
\pi(y, \theta) = U_P(y, t, \theta_P) + U_A(y, t, \theta_A) = u_P(y, \theta_P) + u_A(y, \theta_A).
\] (1)

It is assumed non-negative, strictly concave and increasing up to its maximum \(y^*(\theta) \in \mathcal{Y}\) defined by the unique value satisfying \(\frac{\partial \pi}{\partial y}(y^*(\theta), \theta) = 0\). Given the above assumptions, the profit \(\pi\) and the marginal profit \(\frac{\partial \pi}{\partial y}\) are both increasing with \(\theta_i\) for \(i = P, A\).

The model encompasses for both \(u_P\) and \(u_A\) positive but also for \(u_P\) positive and \(u_A\) negative and reversely. Examples of such functions include firm’s profits, production costs, revenues from marketing a product, as well as utility functions (e.g. benefits from consuming a level of public good \(y\)). For instance, \(u_P(y, \theta_P)\) might stand for the revenue from marketing \(y\) units with a demand level \(\theta_P\) (e.g. \(u_P = (\theta_P - y)y\)) or a price \(\theta_P\) (then \(u_P(y, \theta_P) = \theta_P y\)) while \(u_A(y, \theta_A) = -c(y, \theta_A)\) where \(c\) is a production cost function with marginal cost decreasing with \(\theta_A\).

Three stages can be defined to evaluate player’s payoffs: \textit{ex ante}, before players have received any private information; \textit{interim}, when each player \(i\) has received his or her private information \(\theta_i\) but does not know the other’s information; \textit{ex post}, when the information state \(\theta\) is public. The corresponding ex ante, interim and ex post payoffs are, respectively, \(E_\theta[U_i(y(\theta), t(\theta), \theta_i)]\), \(E_{\theta_j}[U_i(y(\theta), t(\theta), \theta_i)]\), and \(U_i(y(\theta), t(\theta), \theta_i)\) for \(i = P, A, j \neq i\).

An allocation rule \(\{y(\theta), t(\theta)\}_{\theta \in \Theta}\) is a menu of allocations \((y(\theta), t(\theta))\) contingent on each state of nature \(\theta \in \Theta\).

\(^8\)The terminology is similar to Homström and Maskin [3]. \(E_{\theta_j} (E_\theta)\) denotes the expectation operator over \(\theta_j \ (\theta)\).
Definition 1 The allocation rule \( \{ y(\theta), t(\theta) \}_{\theta \in \Theta} \) is Bayesian incentive-compatible (BIC) for \( i \) if
\[
\theta_i \in \arg \max_{\tilde{\theta}_i} E_{\theta_j}[U_i(y(\tilde{\theta}_i, \theta_j), t(\tilde{\theta}_i, \theta_j), \theta_i)]
\]
for every \( \theta_i \in \Theta_i, \theta_j \in \Theta_j, \) and for \( j \neq i \).

Definition 2 The allocation rule \( \{ y(\theta), t(\theta) \}_{\theta \in \Theta} \) is dominant strategy incentive-compatible (DSIC) for \( i \) if
\[
\theta_i \in \arg \max_{\tilde{\theta}_i} U_i(y(\tilde{\theta}_i, \theta_j), t(\tilde{\theta}_i, \theta_j), \theta_i)
\]
for every \( \theta_i \in \Theta_i, \theta_j \in \Theta_j, \) and for \( j \neq i \).

The conditions in Definition 1 (Definition 2) are the standard incentive-compatible constraints that allocation rules must satisfy in any direct revelation mechanisms in a Bayesian (dominant) strategy equilibrium. Bayesian incentive-compatibility (dominant strategy incentive-compatibility) requires that truthful reporting private information maximizes the player’s interim (ex post) payoff. Of course, dominant strategy incentive-compatibility is a stronger requirement in the sense that any DSIC allocation rule is BIC for \( i \) but the reverse is not necessarily true.

In the rest of the paper, I will often use the following equivalent formulations of Bayesian incentive-compatibility and dominant strategy incentive compatibility.

**Lemma 1** An allocation rule is BIC for \( i \) if and only if:

(i) \( E_{\theta_j}[y(., \theta_j)] \) is non-decreasing,

(ii) \( E_{\theta_j}[U_i(y(\theta), t(\theta), \theta_i)] = \int_{\theta_i}^{\theta_j} E_{\theta_i}[(\partial u_i/\partial \theta_i)(y(x, \theta_j), x)]dx + E_{\theta_j}[U_i(y(\theta_i, \theta_j), t(\theta_i, \theta_j), \theta_i)] \),

for every \( \theta_i \in \Theta_i, \theta_j \in \Theta_j, \) and for \( j \neq i \).

**Lemma 2** An allocation rule is DSIC for \( i \) if and only if:

(i) \( y(., \theta_j) \) is non-decreasing,

(ii) \( U_i(y(\theta), t(\theta), \theta_i) = \int_{\theta_i}^{\theta_j} \partial u_i/\partial \theta_i(y(x, \theta_j), x)dx + U_i(y(\theta_i, \theta_j), t(\theta_i, \theta_j), \theta_i) \),
for every $\theta_i \in \Theta_i$, $\theta_j \in \Theta_j$, and for $j \neq i$.

Lemmata 1 and 2 set out well-known results in mechanism design, mostly due to Mirrless [6], in the context of the present model. Formal proofs can be found in Fudenberg and Tirole [2] and Mas-Colell et al. [4]. Although the second conditions in the above lemma looks a bit complex, they have a simple interpretation. Condition (ii) in Lemma 1 (Lemma 2) decomposes $i$’s interim (ex post) payoff into two terms. The second right-hand term is $i$’s interim (ex post) payoff when of type $\theta_i$. The first right-hand term is the incremental interim (ex post) benefit of a higher type $\theta_i > \theta_i$. Condition (ii) tells that $i$ must receive exactly the incremental benefit of reporting truthfully $\theta_i$.

**Definition 3** An allocation rule satisfy ex post participation for $i$ if and only if $i$’s ex post payoffs are non-negative, i.e., $U_i(y(\theta), t(\theta), \theta_i) \geq 0$ for every $\theta \in \Theta$.

An incentive allocation rule is an allocation rule that is BIC for both players and satisfy ex post participation for $A$. This paper examines the equivalent implementation of incentive allocation rules with sequential communication in the sense defined below.

**Definition 4** An incentive allocation rule can equivalently be implemented in a sequential mechanism (or with sequential communication) in which $i$ communicates first if it is:

(i) BIC for $i$,

(ii) DSIC for $j \neq i$,

(iii) it satisfies $A$’s ex-post participation constraints,

(iv) it yields same ex ante payoff to both players.

To be equivalently implemented in a sequential mechanism, an incentive allocation rule must satisfy the stronger requirement of dominant strategy incentive-compatibility.
for one player. The sequence of communication determines the identity of this player. Once $i$ has communicated honestly her or his type, $j \neq i$ selects her or his reporting strategy under perfect information. Therefore, $j$’s incentive-compatible constraints must be satisfied ex post in every state of nature, i.e., in dominant strategy.

I first focus on specific incentive allocation rules: those that are implemented in the informed principal-agent game defined in the next section. I examine more general incentive allocation rules in Section 4.

3 Sequential communication in the informed principal-agent game

Consider the following benchmark game. Suppose that the principal makes ex ante a take-it-or-leave-it incentive allocation rule offer to the agent. If it is refused, each player gets zero. If it is accepted, the game proceeds. Each player $i$ observes privately $\theta_i$. Players simultaneously send direct messages $\hat{\theta}_i$ which select a single allocation $(y(\hat{\theta}_P, \hat{\theta}_A), t(\hat{\theta}_P, \hat{\theta}_A))$ in the incentive allocation rule.

The incentive allocation rules implemented in a Perfect Bayesian Nash Equilibrium of the benchmark game, denoted $\{y^P(\theta), t^P(\theta)\}_{\theta \in \Theta}$, are those who maximizes the principal’s ex ante payoff. Although several transfer rules can be solution, the decision rule $\{y^P(\theta)\}_{\theta \in \Theta}$ is unique and easy to characterize under some assumptions. Applying the expectation operator on $\theta_A$ and integrating by part shows that the agent’s BIC constraints as defined in Lemma 1 implies

$$E_\theta \left[ \frac{1}{f_A(\theta_A)} \frac{\partial u_A}{\partial \theta_A}(y(\theta), \theta_A) + U_A(y(\theta_P, \theta_A), t(\theta_P, \theta_A), \theta_A) \right].$$

The agent’s ex ante payoff is decomposed into two terms. The first term is his expected informational rent. It is the minimal amount that provides him with incentives to communicate truthfully. The second term is the agent of type $\theta_A$’s interim payoff. The principal equalizes this term to zero to maximize her ex ante payoff while satisfying the BIC and ex post participation constraints. Therefore, the agent’s ex ante payoff is just the first term. To simplify notation, denote $r_A(y, \theta_A) =$
\[
\frac{1 - F_A(\theta_A)}{f_A(\theta_A)} \partial u_A(y, \theta_A) \text{ so that the agent’s ex ante payoff writes } E_\theta[r_A(y(\theta), \theta_A)]. \text{ Due to (1), the principal obtains:}
\]

\[
E_\theta[U_P(y(\theta), t^P(\theta), \theta_P)] = E_\theta[\pi(y(\theta), \theta) - r_A(y(\theta), \theta_A)]. \tag{2}
\]

The decision rule implemented \( \{y^P(\theta)\}_{\theta \in \Theta} \) maximizes (2). Under some assumptions (specified later), it is uniquely defined by the following first order conditions:

\[
\frac{\partial \pi}{\partial y}(y^P(\theta), \theta) - \frac{\partial r_A}{\partial y}(y^P(\theta), \theta_A) = 0, \tag{3}
\]

for all \( \theta \in \Theta \). The decision scheme \( \{y^P(\theta)\}_{\theta \in \Theta} \) solves a trade-off between maximizing the total surplus and minimizing the agent’s informational rent. As a consequence, except “at the top” (for the agent), decisions are distorted downward: \( y^P(\theta) < y^*(\theta) \) for all \( (\theta_P, \theta_A) \) such that \( \theta_A \neq \bar{\theta}_A \). The second-order conditions are satisfied under the following assumption.

**Assumption 2** \( \frac{\partial^3 u_A}{\partial y^2 \partial \theta_A} \geq 0. \)

Totally differentiating (3) with respect to \( \theta_A \) shows that the requirement of \( y^P(\theta_P, .) \) non-decreasing holds under the following supplementary assumptions.\(^9\)

**Assumption 3** \( \frac{\partial^2 u_A}{\partial y \partial \theta_A} \leq 0. \)

**Assumption 4** \( \frac{d}{d\theta_A} \left[ \frac{1 - F_A(\theta_A)}{f_A(\theta_A)} \right] \leq 0. \)

Assumption 4 tells that the “hazard rate” \( \frac{f_A(\theta_A)}{1 - F_A(\theta_A)} \) is non-decreasing. It is satisfied for instance if the density \( f_A \) is non-decreasing. Assumptions 1 to 4 are standard in mechanism design (e.g. Fudenberg and Tirole [2], Chapter 7). Armed with those assumptions, we set out a first result.

**Proposition 1** Under assumptions 1 to 4, the incentive allocation rule implemented in the informed principal-agent game is equivalently implementable in a sequential mechanism if and only if the principal communicates first.

\(^9\)Both assumptions imply that \( \frac{\partial^2 r_A}{\partial y \partial \theta_A}(y^P(\theta), \theta_A) \leq 0 \) which insures that \( y^P(\theta_P, .) \) is increasing. Assumptions 1 and 2 and the concavity of \( \pi(., \theta) \) imply that \( y^P(., \theta_A) \) is increasing.
Proof (If part) Consider the following transfer scheme:

\[ \hat{t}(\theta) = -u_A(y^p(\theta), \theta_A) + \int_{\theta_A}^{\theta} \frac{\partial u_A}{\partial \theta_A}(y^p(\theta, x), x)dx. \]

It yields \( U_A(y^p(\theta), \hat{t}(\theta), \theta_A) = \int_{\theta_A}^{\theta} \frac{\partial u_A}{\partial \theta_A}(y^p(\theta, x), x)dx \geq 0 \) to the agent in state \( \theta \) so that ex post participation constraints hold. Moreover, since \( y^p(\theta_p, \cdot) \) is non-decreasing for every \( \theta_p \in \Theta_P \), then the agent’s DSIC constraints as defined in Lemma 2 hold. The above transfer scheme yields \( E_{\theta_A}[U_P(y^p(\theta), \hat{t}(\theta), \theta_P)] = E_{\theta_A}[[\pi(y^p(\theta), \theta) - r_A(y^p(\theta), \theta_A)] \right] \) to the principal at the interim stage. Since \( y^p(\theta) \) maximizes \( \pi(y^p(\theta), \theta) - r_A(y^p(\theta), \theta_A) \) with respect to \( \theta_p \) and, therefore, the allocation rule \( \{y^p(\theta), \hat{t}(\theta)^P\}_{\theta \in \Theta} \) is BIC for the principal.

(Only if part) First, I show that any allocation rule that equivalently implements \( \{y^p(\theta), t^p(\theta)\}_{\theta \in \Theta} \) must yields zero ex post payoffs to the agent of type \( \theta_A \). If not the case, if the agent’s ex post payoff is strictly positive in say state \( (\theta_p, \theta_A) \in \Theta \), then, due to the ex post participation constraints, the agent \( \theta_A \)'s interim payoff is also strictly positive. Using the definition of the BIC conditions for the agent in Lemma 1, applying the expectation operator on \( \theta_A \) and integrating by part as before shows that the agent’s ex ante payoff is then strictly higher than \( E_{\theta}[r_A(y(\theta), \theta_A)] \). Therefore, ex ante, the principal obtains strictly less than the maximal value of \( E_{\theta}[\pi(y(\theta), \theta) - r_A(y(\theta), \theta_A)] \) she gets with simultaneous communication.

Second, I show that any incentive allocation that yields zero ex post payoffs to the agent of type \( \theta_A \) violates the principal’s DSIC constraints. Since the agent gets nothing in states \( (\theta_p, \theta_A) \) for every \( \theta_p \in \Theta_P \), the principal obtains the surplus, namely \( \pi(y^p(\theta_p, \theta_A), (\theta_p, \theta_A)) \), in those states. Since \( y^p(\theta_p, \theta_A) \) for every \( \theta_p \in \Theta_P \), then there exists a type \( \theta_A > \theta_A \) such that \( \pi(y^p(\theta_p, \theta_A), (\theta_p, \theta_A)) > \pi(y^p(\theta_p, \theta_A), (\theta_p, \theta_A)) \) for any \( \theta_p \in \Theta_P \), which implies that the principal prefers to send the message \( \hat{\theta}_A \) rather than revealing truthfully \( \theta_A \).\(^{10}\)

\(^{10}\) Notice that the argument holds for a measurable set around \( \theta_A \).

Proposition 1 tells that, in the context of an informed principal-agent game with
an agent ex post participation constrained, there is no loss of efficiency if communication occurs sequentially if and only if the principal communicates first.

In the principal-agent model, the principal gets all surplus net of the agent’s informational rent in expectation not only at the ex ante stage but also the interim stage. She therefore has incentive to select the set of decisions \( \{y^P(\theta_P, \theta_A)\}_{\theta_A \in \Theta} \) that maximizes her ex ante payoff at the interim stage by reporting truthfully \( \theta_P \).

She somehow internalizes at the interim stage the tradeoff between maximizing the expected surplus and minimizing the agent’s expected informational rent that solves the allocation rules at the ex ante stage. On the other hand, to satisfy the agent’s DSIC and ex post participation constraints, the incentive allocation rule must assign to the agent the incremental gain of reporting truthfully \( \theta_A > \tilde{\theta}_A \), formally
\[
\int_{\tilde{\theta}_A}^{\theta_A} \frac{\partial u_A}{\partial \theta_A}(y^P(\theta_P, x), x) dx.
\]
The principal gets the surplus net of this rent. She therefore obtains all the surplus when the agent’s type is \( \theta_A \). Since decisions are distorted downward (and expecting that the agent report truthfully \( \tilde{\theta}_A \)), she prefers to report a higher type.\(^{11}\) Hence, the incentive allocation rule cannot be DSIC for both players while assigning non-negative payoffs to the agent. Yet it can neither be BIC for the agent and DSIC for the principal for the same reason.

As a next step, I further investigate the informed principal-agent game, imposing non-negative payoffs for the principal as well. It is easy to show that the allocation rule \( \{y^P(\theta), \bar{t}(\theta)\}_{\theta \in \Theta} \) defined above assigns non-negative ex post payoffs to the principal if \( u_A \) is concave in \( \theta_A \). Indeed, the principal obtains
\[
U_P(y^P(\theta), \bar{t}(\theta), \theta_P) = \pi(y^P(\theta), \theta) - \int_{\tilde{\theta}_A}^{\theta_A} \frac{\partial u_A}{\partial \theta_A}(y^P(\theta_P, x), x) dx, \tag{4}
\]
in any state \( \theta \). Since \( \pi(y^P(\theta_P, \theta_A), (\theta_P, \tilde{\theta}_A)) \geq 0 \) for every \( \theta_P \in \Theta \), all we have to show is that the right-hand term in (4) is non-decreasing in \( \theta_A \). Differentiate (4) with respect to \( \theta_A \) and use the envelop theorem to compute:
\[
\frac{\partial \pi}{\partial \theta_A}(y^P(\theta), \theta) - \frac{\partial u_A}{\partial \theta_A}(y^P(\theta), \theta_A) - \int_{\tilde{\theta}_A}^{\theta_A} \frac{\partial^2 u_A}{\partial \theta_A^2}(y^P(\theta_P, x), x) dx.
\]
\(^{11}\)If her type is say \( \theta_P' < \tilde{\theta}_P \), she prefers to report \( \theta_P'' > \theta_P' \) (ideally \( \theta_P'' \) such that \( y^P(\theta_P'', \tilde{\theta}_A) = y^*(\theta_P'', \tilde{\theta}_A) \) if it exists) in order to maximize the surplus which is also her payoff.

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Since $\frac{\partial \pi}{\partial \theta}(y^P(\theta), \theta) = \frac{\partial u_A}{\partial \theta_A}(y^P(\theta), \theta_A)$, then the above term is non-negative under the assumption $\frac{\partial^2 u_A}{\partial \theta_A^2}(y^P(\theta), \theta_A) \leq 0$ for every $\theta \in \Theta$.

**Corollary 1** Suppose $u_A(y, \cdot)$ is concave for any $y \in Y$, then Proposition 1 holds with the principal ex post participation constrained as well.

Corollary 1 provides a clear link between communication order and bargaining power when both players are treated similarly regarding ex post participation. It tells that sequential communication performs as well as simultaneous communication if and only if the player who has bargaining power (the principal) moves first.

### 4 Equivalent implementation of any incentive allocation rule with sequential communication

In this section, I examine the equivalent implementation of *any* incentive allocation rule\(^{12}\) in the bilateral asymmetric framework described in Section 2 (not necessarily the ones that maximizes $P$’s ex ante payoff).

It is useful to show that the DSIC condition combined with the ex post participation constraints imposes specific lower bounds on the agent’s payoff. Indeed, applying the expectation operator over $\theta_A$ to condition (ii) in Lemma 2 and integrating by part shows that, to be DSIC for the agent, an allocation rule must satisfy,

$$E_{\theta_A}[U_A(y(\theta), t(\theta), \theta_A)] = E_{\theta_A}[r_A(y(\theta), \theta_A)] + U_A(y(\theta_P, \theta_A), t(\theta_P, \theta_A), \theta_A).$$

The ex post participation constraints impose the second right-hand term to be non-negative, therefore,

$$E_{\theta_A}[U_A(y(\theta), t(\theta), \theta_A)] \geq E_{\theta_A}[r_A(y(\theta), \theta_A)].$$

\(^{12}\)Recall that incentive allocation rules are allocation rules that are BIC for both players and satisfy ex post participation for $A$. 

It turns out that the above inequalities\(^{13}\) for every \(\theta_p \in \Theta_P\) in addition to condition (i) of Lemma 2 are not only necessary but also sufficient conditions for the equivalent implementation of incentive allocation rules when \(P\) moves first.

**Proposition 2** Under assumption 1, any incentive allocation rule \(\{y(\theta), t(\theta)\}_{\theta \in \Theta}\) can be equivalently implemented in a sequential mechanism in which \(P\) communicates first if and only if \(y(\theta_P, .)\) is non-decreasing and \(E_{\theta_A}[U_A(y(\theta), t(\theta), \theta_A)] \geq E_{\theta_A}[r_A(y(\theta), \theta_A)]\) for every \(\theta_P \in \Theta_P\).

**Proof (If part)** Consider an incentive allocation rule \(\{y(\theta), t(\theta)\}_{\theta \in \Theta}\) that satisfies Proposition 2’s conditions. Consider further the following transfer scheme:

\[
t'(\theta) = -u_A(y^p(\theta), \theta_A) + \int_{\Theta_A} \partial u_A(y^p(\theta, x), \theta) dx + \gamma(\theta_P),
\]

with \(\gamma(\theta_P) \equiv E_{\theta_A}[^{\pi}(y(\theta), \theta) - r_A(y(\theta), \theta_A) - U_P(y(\theta), t(\theta), \theta_P)]\) for every \(\theta \in \Theta\).

First, I show that \(\{y(\theta), t'(\theta)\}_{\theta \in \Theta}\) is BIC for \(P\). Apply the expectation operator over \(\theta_A\) and use (1) to compute

\[
E_{\theta_A}[U_P(y(\theta), t'(\theta), \theta_P)] = E_{\theta_A}[\pi(y(\theta), \theta) - \int_{\Theta_A} \partial u_A(y(\theta, x), \theta) dx - \gamma(\theta_P)].
\]

Integrating by part yields

\[
E_{\theta_A}[U_P(y(\theta), t'(\theta), \theta_P)] = E_{\theta_A}[\pi(y(\theta), \theta) - r_A(y(\theta), \theta_A)] - \gamma(\theta_P).
\]

Substitute for \(\gamma(\theta_P)\) and the last relationship simplifies to:

\[
E_{\theta_A}[U_P(y(\theta), t'(\theta), \theta_P)] = E_{\theta_A}[U_P(y(\theta), t(\theta), \theta_P)]. \tag{7}
\]

Therefore, since \(\{y(\theta), t'(\theta)\}_{\theta \in \Theta}\) is BIC for \(P\), so is \(\{y(\theta), t'(\theta)\}_{\theta \in \Theta}\).

Second, \(\{y(\theta), t'(\theta)\}_{\theta \in \Theta}\) is obviously DSIC for \(A\) since, by construction, it satisfies condition (ii) in Lemma 2.

\(^{13}\)Notice that, since \(E_{\theta_A}[U_A(y(\theta), t(\theta), \theta_A)] \geq E_{\theta_A}[r_A(y(\theta), \theta_A)]\) is equivalent to \(E_{\theta_A}[U_P(y(\theta), t(\theta), \theta_P)] \leq E_{\theta_A}[\pi(y(\theta), \theta) - r_A(y(\theta), \theta_A)]\) for every \(\theta_P \in \Theta_P\), it specifies that the first-mover’s expected payoff must be lower than the surplus net of her partner’s informational rent at the interim stage.
Third, A’s ex post payoff in state $\theta \in \Theta$ writes:

$$
\int_{x_A}^{\theta_A} \frac{\partial u_A}{\partial \theta_A}(y(\theta_P, x), x)dx + E_{\theta_A}[\pi(y(\theta), \theta) - r_A(y(\theta), \theta_A) - U_P(y(\theta), t(\theta), \theta_P)].
$$

The first term above is obviously non-negative. The second term is also non-negative because, due to (1),

$$
E_{\theta_A}[\pi(y(\theta_P), \theta) - U_P(y(\theta), t(\theta), \theta_P)] = E_{\theta_A}[U_A(y(\theta), t(\theta), \theta_A)],
$$

which is higher than $E_{\theta_A}[r_A(y(\theta), \theta_A)]$ for every $\theta_P \in \Theta$ by assumption.

Lastly, (7) shows that $\{y(\theta), t'(\theta)\}_{\theta \in \Theta}$ yields same interim payoff and, therefore, same ex ante payoff than $\{y(\theta), t(\theta)\}_{\theta \in \Theta}$ to $P$. Furthermore, (7) combined with (1) yields $E_{\theta_A}[U_A(y(\theta), t'(\theta), \theta_A)] = E_{\theta_A}[U_A(y(\theta), t(\theta), \theta_A)]$ for every $\theta_P \in \Theta$. Therefore, applying the expectation operator over $\theta_P$ shows that $\{y(\theta), t'(\theta)\}_{\theta \in \Theta}$ yields same ex ante payoff than $\{y(\theta), t(\theta)\}_{\theta \in \Theta}$ to $A$.

(Only if part) The necessary conditions follow from condition (i) in Lemma 2 and

the implication of condition (ii) in Lemma 2 established in (6). $\square$

Proposition 2 provides sufficient conditions for the equivalent implementation of incentive allocation rules with a sequential mechanism (without restriction on the order of moves).
References


