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## COOPERATION AND EQUITY IN THE RIVER SHARING PROBLEM

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# Cooperation and Equity in the River Sharing Problem

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## Abstract

This paper considers environments in which several agents (countries, farmers, cities) share water from a river. Each agent enjoys a concave benefit function from consuming water up to a satiation level. Noncooperative extraction is typically inefficient and any group of agents can gain if they agree on how to allocate water with monetary compensations. The paper describes which allocations of water and money are acceptable to riparian agents according to core stability and several criteria of fairness. It reviews some theoretical results. It then discusses the implementation of the proposed allocation with negotiation rules and in water markets. Lastly, it provides some policy insights.

**Keywords:** Water, Game, Core, Fairness, Water Market, Negotiation Rules, Externalities.

**JEL codes:** Q25 (Water), Q28 (Government Policy)

## 1 Introduction

Water is essential to life. It is consumed for a variety of purposes, from domestic to agricultural and industrial uses. Due to population growth, the development of irrigated agriculture, and industrialization, demand has tremendously increased, and water has become a locally scarce resource in many regions on earth. The so-called tragedy of the commons has considerable relevance to water resources: free-access (or decentralized) extraction leads to inefficiencies, and increasing benefits to all users requires centralized planning and cooperation of the economic agents (farmers, firms, cities, countries) who share a water resource. In practice, concretization of such coordination may take many forms, from international agreements signed by sovereign countries to allocation rules or water markets established by communities of farmers. The process is often facilitated by local authorities, in many cases with the involvement of users.

This paper deals with the issue of coordinating water management along rivers. It investigates the incentives of riparian agents to agree to share water efficiently. It examines what kind of agreement is acceptable. The definition of “acceptability” is twofold. First, the river sharing agreement (or allocation) should be stable in the sense that no users or group of users are better off designing another river sharing agreement. Furthermore, it should be perceived as fair according to certain justice principles.

This issue is tackled using cooperative game theory and the axiomatic theory of justice. The paper describes the cooperative game induced by a river sharing problem, and analyzes the stable river sharing agreements in this cooperative game. Next, it considers standard axiomatic principles of fair division and adapts them to the river sharing problem.<sup>1</sup> It posits fair sharing rules for total welfare.

The paper reviews several important theoretical papers on the river sharing problem in an informal and simple way, without formal proofs. The goal is to provide the intuition of these results and their policy implications.

Note that the focus here is on the direct benefits of water consumption. The model ignores other benefits for which water is not directly consumed, such as flood control, navigation, recreation activities, biodiversity, and hydropower production.<sup>2</sup>

Rivers are common water resources that possess several interesting features. First, each agent can only consume water entering the river upstream of its location. Therefore, agents have unequal access to the resource, depending on their location on the river. Upstream agents have a first mover advantage in water extraction. Yet, as river flow increases downstream, this advantage is offset by the lower amount of water available upstream. Second, the welfare achieved by cooperation depends on the locations of the cooperating agents. In order to increase welfare, a group of agents exchanges water. This can only be done from upstream agents to

downstream agents and, preferably, by neighbors: water exchange among distant agents is subject to extraction or free riding by those located in between.

International river sharing agreements are numerous worldwide. For instance, the Nile Treaty, signed in 1929, specifies a sharing rule for the Nile River water flow between Egypt and Sudan. The Columbia River Treaty specifies a sharing rule for the costs and benefits from flood control and hydropower production between Canada and the United States (Barrett 1994). In the case of the Syr Darya River, the upstream country, Kyrgyzstan, agreed to increase summer discharges to supply the downstream country, Uzbekistan, in exchange for fossil fuel transfers (Abbink, Moller, and O'Hara 2005). Similarly, Laos and Thailand signed an agreement on developing hydropower production on one of the tributaries of the Mekong River tributaries inside the former country. It specifies a payment in hard currency from Thailand to Laos in exchange for electricity supply (Barrett 1994). In the United States, states sign interstate river compacts that prescribe a fixed or a percentage allocation of water (Bennett, Howe, and Shope 2000).

Governments or farmers themselves set up rules to encourage efficient exploitation of water for irrigation, including water pricing, subsidies, and water markets (Ostrom 1990; Dinar, Rosegrant, and Meinzen-Dick 1997). Such rules lead to an allocation of water and can result in a redistribution of the benefits arising from water extraction. Because different users (for example farmers, urban dwellers) use water for different purposes and thus derive different values from an additional unit of water, there is an impetus for moving water from lower-value to higher-value uses. During this process, the seller of a certain volume of water obtains monetary compensation from those who buy it. In general, farmers have to pay for water consumption. The money collected is then spent on maintenance or transferred to some users through subsidies. For instance, Thomas, Feres, and Nauges (2004) provide evidence that French farmers receive on average four times more subsidy than the amount they pay to the water agencies.

These monetary transfers, whether they are direct (peer-to-peer in a water market) or not (through centralized water pricing, taxes, or subsidies), and the allocation of water comprise the total benefit from consuming water, defining a particular distribution of the total welfare. When water is exclusively consumed to irrigate crops, the farmer's total welfare is simply the value of total production, though it could include the monetary equivalent of the utility consumers derive from consuming water.

The paper proceeds as follows. Section 2 introduces a general model to address the issue of cooperation and equity in river sharing. Section 3 examines the optimal allocation of water in this model. As shown in section 4, noncooperative free-access extraction leads to an inefficient allocation of water. Therefore, any movement towards Pareto optimality requires cooperation and monetary compensation mechanisms that are acceptable to all agents. The transfers are only acceptable if the allocation of water and money belongs to the core of the cooperative game (section 5) or is perceived as fair (section 6). Section 7 presents a negotiation game that lead to

an efficient allocation of water and fair and stable transfer schemes in the subgame perfect equilibrium. Section 8 posits property rights that lead to efficient water allocation and fair transfers in competitive water markets. The concluding section discusses the policy implications of the analysis of the river sharing problem.

## 2 General framework

The river sharing problem is represented by the following stylized model. Agents are ranked according to their location along the river and numbered from upstream to downstream:  $i < j$  means that  $i$  is upstream of  $j$  and  $j$  is downstream of  $i$ . There are  $n$  agents. The set of agents is denoted by  $N = \{1, \dots, n\}$ . Agent  $i$ 's benefit or production function from consuming a level  $x_i$  of water is  $b_i(x_i)$ . Benefits are measured in monetary terms. The function  $b_i$  is strictly concave and differentiable for every  $x_i$  and  $i \in N$ . It is increasing up to a satiation level  $y_i$ . Above this level, the agent infers a loss from overconsumption. Indeed, above satiation, the cost of extraction and sanitation exceeds the benefit from consumption; or, even worse, the agent suffers from flooding. Marginal benefits are strictly decreasing and positive up to the satiation level. Above that, they are strictly decreasing and negative. It is also assumed that the marginal benefit at 0 (no water) is high enough to avoid corner-type solutions (no water consumption by some agents) in the efficient water allocation program. This means that water is indispensable for a user, as it is very costly (or deadly) to be deprived of water. Examples of such benefit functions are quadratic forms such as  $b_i(x_i) = a_i x_i - b_i \frac{x_i^2}{2}$  where  $y_i = \frac{a_i}{b_i}$  providing that the marginal benefit at zero  $a_i$

is high enough so that everybody should be supplied with water. Our assumptions on the benefit functions are broadly consistent with the production function for irrigated crops, as represented, for example, by Griffin (2006, p.19), except for the marginal benefit at 0.

The amount of water entering the river at the location of the most upstream agent 1 is  $e_1 > 0$ . In addition, several tributaries might enter the river after the location of agent 1. Denote by  $e_i \geq 0$  the amount of water flowing from tributaries located between agents  $i-1$  and  $i$  for  $i=2, \dots, n$ . The highest total amount of water available at location  $i$  (without extraction upstream) is thus  $E_i = e_1 + e_2 + \dots + e_{i-1} + e_i > 0$ . A river sharing problem is formally defined by a set of agents  $N$ , a vector of (strictly concave and single-peaked) benefit functions  $(b_1, \dots, b_n)$ , and a vector of water inflows along the river  $(e_1, \dots, e_n)$ .

## 3 Noncooperative extraction

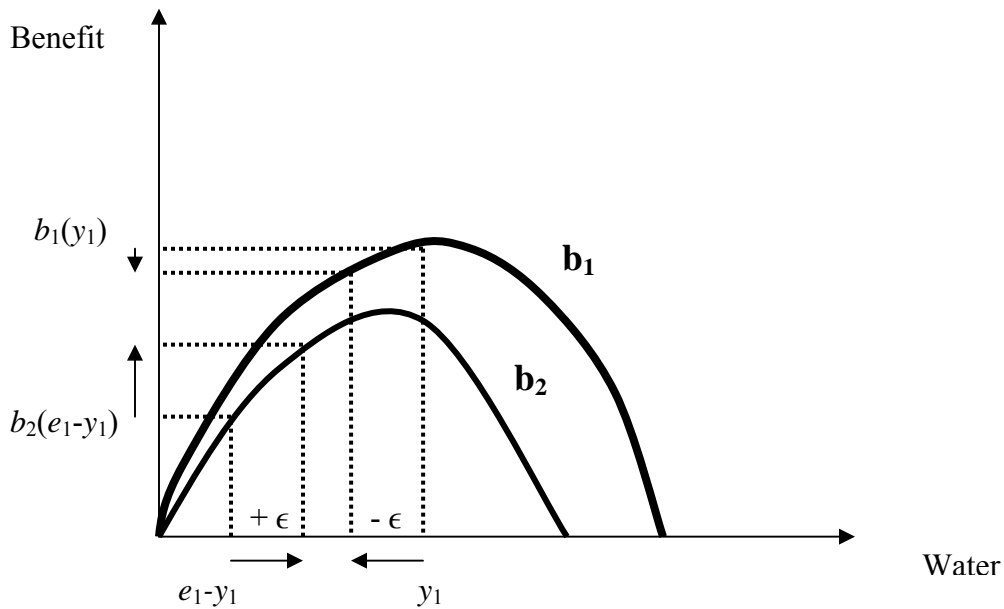
Under noncooperative extraction, the river sharing problem defines the following sequential game. Player 1 chooses how much to consume  $x_1$  under the constraint that this level does not exceed the amount available  $e_1$ . Then player 2 selects  $x_2$  from the remaining water  $e_1 - x_1 + e_2$ , that

is the water left by the upstream agent 1 added to the water flowing between 1 and 2. And so forth until agent  $n$ . Of course, guided by selfishness, each player  $i$  maximizes its benefit function  $b_i$  when choosing  $x_i$  subject to the constraint  $x_i \leq e_1 - x_1 + \dots + e_{i-1} - x_{i-1} + e_i$ .

In the subgame perfect equilibrium of this game, each player  $i$  extracts the maximum between its satiation level  $y_i$  and the amount of water available at its location  $e_1 - x_1 + \dots + e_{i-1} - x_{i-1} + e_i$ . The remaining water is left in the river to be consumed by the downstream agents.

The noncooperative equilibrium is in general inefficient in the sense that the payoff of players can be increased with another allocation of water through monetary compensations. In other words, there is often room for Pareto improvement by allowing transfers between agents.

To understand that, consider simply a river shared by two agents ( $n=2$ ) with no tributaries ( $e_2=0$ ). The upstream agent 1 consumes the minimum of  $y_1$  and  $e_1$ , enjoying a benefit  $b_1(\min\{e_1, y_1\})$ . If  $e_1 \leq y_1$  then the downstream agent 2 gets nothing. If  $e_1 > y_1$  and  $e_1 - y_1 < y_2$ , agent 2 consumes  $e_1 - y_1$  and enjoys  $b_2(e_1 - y_1)$ . This case is represented in Figure 1.



**Figure 1 Example of inefficient noncooperative extraction**

Now reduce the upstream agent's extraction by  $\epsilon$ , as in Figure 1, to increase the downstream agent's consumption by the same amount, thus benefiting the latter. Due to the strict concavity of the benefit functions (or due to diminishing marginal benefits), the increase in benefit to the

downstream agent is larger than the loss of benefit to the upstream agent. Formally, for  $\epsilon > 0$  small enough  $b_2(e_1 - y_1 + \epsilon) - b_2(e_1 - y_1) > b_1(y_1) - b_1(y_1 - \epsilon)$ . In Figure 1, the vertical downward arrow is shorter than the vertical upward arrow. Of course, the same argument applies if the upstream agent consumes  $e_1$  so there is no water left downstream (and also for other assumptions on the number of agents and water inflows). This change of water allocation is Pareto improving if the upstream agent is somehow compensated by the downstream agent for its loss of benefit. It therefore requires some sort of transfer from agent 2 to agent 1. But which transfer?

With two agents, the problem is easy. Let us denote  $(x_1^*, x_2^*)$  the efficient allocation of water (where  $x_i^*$  stands for the water consumed by agent  $i$ ), which, by definition, maximizes the sum of the two agent's benefits  $b_1(x_1) + b_2(x_2)$ , subject to the resource constraint  $x_1 + x_2 \leq e_1$ . The two parties will agree to change their water consumption to  $(x_1^*, x_2^*)$ , providing that 2 pays a transfer  $t$  to 1, if they are both better off doing so. The upstream agent 1 accepts if  $b_1(x_1^*) + t \geq b_1(y_1)$  while downstream agent 2 agrees if  $b_2(x_2^*) - t \geq b_2(e_1 - y_1)$ . The transfer must therefore satisfy the two acceptability constraints  $b_2(x_2^*) - b_2(e_1 - y_1) \geq t \geq b_1(y_1) - b_1(x_1^*)$ . With more than two agents, the transfers need to be acceptable not only to individual agents but also to any group (or coalition) of agents. Otherwise, a group of agents is better off by refusing the agreement and signing its own agreement on the part of the river it controls. Technically speaking, the transfer scheme defines a distribution of welfare. To be acceptable, this distribution of welfare should be in the core of the cooperative game in the sense that no coalition of agents is better off by forming its own river sharing agreement.

Furthermore, the transfer should be perceived as fair by riparian agents. Indeed fairness is often invoked as the main principle for sharing life-essential natural resources such as water. Several fair ways to share the benefit of river water extraction will be considered.

This paper focuses on transfer schemes that implement efficient allocations of water while being acceptable to  $n$  riparian agents (in ways to be defined) for any  $n > 1$  (for example more than two agents). It then discusses how to implement them. First, the optimal allocation of water in the river sharing problem is analyzed.

#### 4 Efficient extraction

To simplify the analysis without loss of generality, assume  $e_i \leq y_i$  for every  $i$ . Indeed, if  $e_i > y_i$  for one  $i$ , then since agent  $i$  will never choose to consume more than  $y_i$ , the next downstream agent  $i+1$  can rely on  $e'_{i+1} = e_{i+1} + e_i - y_i$ . So the river sharing problem can always be redefined with water supplies  $e'_i = y_i$  and  $e'_{i+1} = e_{i+1} + e_i - y_i$  and so forth.

The efficient allocation of water is the allocation that maximizes the total benefit from water extraction. Formally, it is the vector  $\mathbf{x}=(x_1,\dots,x_n)$  that maximizes the sum of all  $b_i$ , that is,  $b_1(x_1)+\dots+b_n(x_n)$ . Such an allocation should be feasible in the sense that what is extracted up to any location  $i$  should not exceed what is available up to that point, that is,  $x_1+\dots+x_i \leq E_i=e_1+\dots+e_i$  for  $i=1,\dots,n$ .

The above feasibility conditions define  $n$  resource constraints. The solution to this problem is formally described in Ambec and Sprumont 2002 and Kilgour and Dinar 2001. An informal description is provided here.

Basically, the efficient allocation divides the set of agents  $N$  into  $K$  subsets of consecutive agents  $N_1=\{1,\dots,i\}$ ,  $N_2=\{i+1,\dots,j\},\dots,N_K=\{h+1,\dots,n\}$ . In a given subset  $N_k$ , the allocation of water equalizes marginal benefits among all members. Across subsets, the marginal benefits decrease. Agents consuming their satiation level have marginal benefit 0 and they must belong to  $N_K$ . The intuition is as follows. Since the marginal benefits are decreasing, the efficient way to share the total amount of water flowing down the river  $E_n=e_1+\dots+e_n$  is to equalize marginal benefits across agents when possible. This marginal benefit is positive as soon as water becomes scarce (that is, if not everybody can consume its satiated level  $y_i$ ) and is equal to the shadow cost of the water.

In the special case of all water inflows coming from the source  $e_1$  (i.e.,  $e_2=e_3=\dots=e_n=0$  or  $e_1=E_n$ ) and with the same benefit functions, the total amount of water  $e_1$  is optimally shared equally among all users, each of them getting  $\frac{e_1}{n}$ . But, in general, agents are heterogeneous. Then those with higher marginal benefits (for example the more-productive farmers) obtain more than the others. Still, with only one source of water  $e_1$ , the water is shared such that the marginal benefits of all agents in the river are equal to the shadow cost of the unique resource constraint  $x_1+\dots+x_n=e_1$ .

Now, if the water picks up volume along its course, there is more water downstream than upstream. The total amount of water available at the downstream end  $E_n$  is efficiently shared among riparian agents if marginal benefits are equalized. The condition might not be feasible due to the lack of water at some location along the river. At say location  $j$ , there might not be enough water to achieve this goal. For instance, if agents are endowed with identical benefits, it might not be possible to assign  $\frac{E_n}{n}$  to agent  $j$  and, therefore, to those upstream of  $j$ . This means that water is more scarce at  $j$  than downstream. Therefore the shadow value is higher. Then it is efficient to make the agents upstream of  $j$ , including  $j$ , share the total inflow up to  $j$   $E_j=e_1+\dots+e_j$ , those downstream of  $j$  relying on the water flowing from tributaries  $j+1,\dots,n$ . This defines the subset  $N_K$  in which agents' marginal benefits are equal. Again, efficiency prescribes that  $E_j$  is shared so as to equalize marginal benefit among agents  $1,\dots,j$ . But this might not be feasible at

say  $i < j$ , in which case marginal benefits are equalized among agents  $i$  and  $j$  and all others in between. This defines the subset  $N_{k-1}$ . And so on until the source of the river is reached.

To sum up, the efficient allocation defines the subset of consecutive agents or portions of the river  $N_k$  in which the total water inflow is shared so as to equalize marginal benefits among agents and to the shadow value of water. This shadow value decreases strictly moving downstream from one portion of river  $N_k$  to the next one  $N_{k+1}$ .

For instance, suppose that  $n=3$ ,  $b_1(x) = 20x - x^2 = b_3(x)$ , and  $b_2(x) = 8x - x^2$ . The satiated consumption levels are  $y_1=y_3=10$  and  $y_2=4$ . Suppose first that  $(e_1, e_2, e_3) = (9, 3, 6)$ . Then efficiency is achieved when the total amount of water  $e_1 + e_2 + e_3 = 18$  is divided such as to equalize marginal benefits among agents. Formally,  $(x_1^*, x_2^*, x_3^*)$  satisfies  $20 - 2x_1^* = 8 - 2x_2^* = 20 - 2x_3^*$  and the binding resource constraint at the end of the river  $x_1^* + x_2^* + x_3^* = 18$ . This leads to  $x_1^* = x_3^* = 8$  and  $x_2^* = 2$ . The shadow cost of the resource is then 4 at any location on the river. Now suppose that  $(e_1, e_2, e_3) = (6, 4, 8)$ , that is, most water flow comes from a downstream tributary and not the source, but still the total amount of water to be shared is  $e_1 + e_2 + e_3 = 18$ . Then agent 1 cannot consume 8 units of water so as to equalize all marginal benefits. The resource constraint at that agent's location, formally  $x_1 \leq e_1$ , is binding and therefore  $x_1^* = e_1 = 6$ . The next two agents share the rest of the water flow  $e_2 + e_3 = 4 + 8 = 12 = x_2^* + x_3^*$  so as to equalize marginal benefits, that is,  $8 - 2x_2^* = 20 - 2x_3^*$ , which leads to  $x_2^* = 3$  and  $x_3^* = 9$ . Therefore the set of agents  $N$  is divided into two subsets  $N_1 = \{1\}$  and  $N_2 = \{2, 3\}$  whose agents share the water they control. The marginal benefit of agent 1 in  $N_1$  is 8. This is the shadow value of water at its location, and is higher than the marginal benefits of agents 2 and 3 in  $N_2$ , or the shadow value of water downstream, which is 2.

## 5 Cooperation

The efficient allocation of water  $\mathbf{x}^* = (x_1^*, \dots, x_n^*)$  described above generally requires that upstream users refrain from extracting water from the river. If nobody (for example a regulator) can oblige them to act thus, they will accept only if they receive (monetary) compensation, which should at least cover the loss from consuming less water than available. But, since such compensation would be financed by downstream agents, it should not exceed the gain in benefit of the downstream agents. In the two-agent example in Figure 1, the upstream agent would accept a reduction in consumption of  $\epsilon$  if compensated by at least the amount represented by the vertical downward arrow. On the other hand, the maximum compensation of the upstream agent acceptable to the downstream agent is represented by the vertical upward arrow. With more than two agents, the arguments should apply not only to individuals, but also to groups. Any group of

agents should be compensated at least for the total of the amounts that the agents could obtain if operating individually. These acceptability conditions of transfers, which implement the efficient allocation  $x^*$ , are based on the notion of the core in cooperative game theory. They require defining formally what a group of agents can achieve by itself in the river sharing problem – the value or characteristic function, in the jargon of cooperative game theory.

Consider a group of agents or coalition  $S$ . The total benefit or welfare that a coalition can achieve alone is called the “value” of that coalition. In the river sharing problem, it is the total benefit of the best way to share the water the coalition can rely on. In other words, it is the group’s benefit from the efficient allocation of water available. Yet what is available to a coalition still has to be defined. A coalition can, of course, rely on the flow from tributaries they control, or  $e_i$  for every  $i \in S$ . But they might also receive inflow of water from outside the coalition. The amount of external water flow depends on how the agents outside the coalition behave, in particular whether they cooperate (by forming a coalition) or not (by playing noncooperatively). To understand that, consider a river shared by three agents. Consider the middle agent 2, located between 1 and 3. How much water can 2 expect to rely on at its location? It will depend on whether the other two agents 1 and 3 cooperate or not. If they do not, 1 consumes the maximum of the water inflows  $e_1$  in the river at its location, with a satiation consumption  $y_1$ . This maximum is equal to  $e_1$  (recall that by assumption  $e_i \leq y_i$  for every  $i$ ). Thus the amount of water available at 2’s location is just the inflow from the tributary it controls  $e_2$ . The maximal benefit that 2 achieves is therefore  $b_2(e_2)$  when agents 1 and 3 act noncooperatively, that is, when they belong to different coalitions. Now, if 1 and 3 group together, then it might be in their interest to pass some water from 1 to 3 through 2 (even if 2 consumes part of this water flow). This is particularly likely if there are no tributaries between 2 and 3 ( $e_3=0$ ), when the most downstream agent 3 can rely on is water left in the river by upstream agents 1 and 2. Agent 3 thus consumes 0 if agent 1 does not leave any water running down the river (which is very costly or deadly). But if agent 1 leaves a sufficient amount of water, at least more than  $y_2 - e_2$ , then even if 2 consumes up to its satiation level  $y_2$  (which it would do), 3 enjoys a positive consumption level of water. The marginal benefit of these first units of water being high, 3 might overcome the loss of benefit by 1 even if there is some water lost to agent 2. Therefore, passing some water between 1 and 3 sometimes increases the total benefit of  $\{1,3\}$  even if it is at the cost of supplying 2, an outsider of  $\{1,3\}$ . Then 2 consumes its satiation level  $y_2$  and obtains its highest benefit  $b_2(y_2)$ , while the coalition  $\{1,3\}$  loses  $y_2 - e_2$  from  $e_1$  in this transfer.

The above argument can easily be illustrated with the example introduced in section 6.4. Suppose  $n=3$ ,  $b_1(x) = 20x - x^2 = b_3(x)$ ,  $b_2(x) = 8x - x^2$  and  $(e_1, e_2, e_3) = (9, 3, 6)$ . Consider agent 2. How much water can it expect to rely on at its location? What benefit can it achieve on its own? If agents 1 and 3 do not cooperate, then agent 1 consumes  $e_1=9$  units of water flow coming from the source and, therefore, agent 2 relies only on  $e_2=3$  units, enjoying a benefit of  $b_2(e_2)=b_2(3)=15$ . If agents 1 and 3 do cooperate, in other words if 1 and 3 are in the same coalition, then agent 1

might supply agent 3 with some of its 9 units of water. In this case, agent 2 would extract the water flowing down at its location up to its satiated level  $y_2=4$ . Since  $e_2=3$ , it would consume up to 1 unit of the water transferred by agent 1 to agent 3. To reach agent 3, the water flow left by agent 1 should therefore exceed 1 unit. By transferring water downstream, the coalition  $\{1,3\}$  loses  $y_2-e_2=1$  unit of water and, therefore, can rely on  $e_1+e_3-1=9+6-1=14$  units of water. Since they have the same benefit function, the efficient way to share these 14 units is to divide the amount equally  $x_1=x_3=7$  (which is feasible because  $7 \leq e_1$ ). In doing so, the coalition  $\{1,3\}$  enjoys a benefit of  $b_1(7)+b_3(7)=196$ , which is higher than its benefit without transferring water, which is  $b_1(e_1)+b_3(e_3)=b_1(9)+b_3(6)=99+84=183 < 196$ . Therefore, despite the loss of 1 unit of water, if agents 1 and 3 cooperate, agent 1 leaves 3 units of water flowing down the river but only 2 units reach agent 3. Agent 2 consumes its satiated level  $y_2=4$ , enjoying a benefit of  $b_2(4)=16$ , which is strictly more than the 15 units it gets if 1 and 3 do not cooperate.

As shown above, the maximal benefit or value of a coalition  $S$  depends on the coalition structure of the other agents  $N \setminus S$ .<sup>3</sup> At one extreme, all members outside  $S$  can act cooperatively by forming a single coalition  $N \setminus S$ . Similarly, as above, they might pass some water through some agents that belong to  $S$ . At the other extreme, all members outside  $S$  form singletons. They act noncooperatively and thus pass no water through subsets of  $S$ . Between those two extremes, one can think of other more or less coarse coalition structures or partitions of  $N \setminus S$ . In general, a partition of  $N$  defines a sequential game in which agents cooperate within coalitions but not between coalitions. Broadly speaking, such a game is similar to that described in the noncooperative extraction section (6.3) except that the players are consecutive subcoalitions.<sup>4,5</sup> Moreover, those who belong to the same coalition cooperate while the others do not. This game is formally described and analyzed in Ambec and Ehlers 2006.

As suggested by the above example and proved in Ambec and Ehlers 2006, the value of a coalition is higher if outside members cooperate than if they do not. The basic idea is that if people outside  $S$  cooperate they might pass some water through members of  $S$ , while they are unlikely to do so if they act noncooperatively. When computing how much welfare they can achieve by their own, the members of a coalition must have expectations about how the others will behave. They might thus expect to get some water from outside the part of the river they control. The pessimistic view is that outsiders do not cooperate at all. They form singletons in the partition of the sequential game. They thus never leave any water from their inflows to downstream agents, including members of  $S$  (recall that we have normalized to  $e_i \leq y_i$  but they might leave what exceeds their peak consumption  $y_i$  if the amount of water coming from the upstream agents is sufficiently large). Call the value function the “cooperative value” when members outside cooperate; and the “noncooperative value” when they do not cooperate.<sup>6</sup>

Going back to the problem of searching for acceptable contributions and compensations, denote by  $t=(t_1, \dots, t_n)$  the allocation of “money” or transfer scheme, where  $t_i$  denotes the compensation assigned to agent  $i$  (which is negative in the case of a contribution). The allocation should be

budget balanced: it must sum up to 0 or less. The transfer scheme  $t$  and the water allocation  $x^*$  yield a payoff or utility  $b_i(x_i^*) + t_i$  to any agent  $i$  for every  $i \in N$ . A transfer scheme defines a distribution of the maximal total welfare. The transfer scheme  $t$  or the allocation  $(x^*, t)$  is acceptable in the sense of the core if every group of agents  $S$  obtains at least what it can achieve on its own, that is, its value. Formally, the sum of the payoffs  $b_i(x_i^*) + t_i$  of the agents belonging to  $S$  is at least  $v(S)$  for any  $S \subset N$ . The core defines the lower bounds on agents' payoffs or, equivalently, on transfers  $t_i$  (given the optimal allocation of water  $x^*$ ). They depend on the value function  $v$  under consideration. The cooperative (or noncooperative) core lower bounds are the ones defined using the cooperative (or noncooperative) values of coalitions.

Since the cooperative value characteristic function is greater than or equal to the noncooperative function, the noncooperative lower bounds are easier to satisfy. Ambec and Ehlers (2006) show that, in any river sharing problem, the transfer scheme that assigns to every agent its marginal contribution to its predecessors satisfies the noncooperative core lower bounds, yielding a so-called downstream incremental distribution. Formally, denoting  $Pi = \{1, \dots, i\}$  the set of predecessors of  $i$  (including  $i$ ) and  $P^\circ i = \{1, \dots, i-1\}$  the set of strict predecessors of  $i$ , the downstream incremental distribution assigns to any agent  $i$  the payoff  $b_i(x_i^*) + t_i = v(Pi) - v(P^\circ i)$ .

It thus prescribes a transfer scheme  $t^d$  with  $t_i^d = -b_i(x_i^*) + v(Pi) - v(P^\circ i)$  for every  $i \in N$ . Furthermore, other transfer schemes might also satisfy the noncooperative core lower bounds. In the particular case where benefit functions are always increasing (so  $y_i$  goes to infinity), Ambec and Sprumont (2000) show that then the cooperative game is convex. This implies that the noncooperative core lower bounds might be satisfied by many transfer schemes, including the one that assigns to any agent its marginal contribution to the coalition composed of its followers  $v(Fi) - v(F^\circ i)$ , where  $Fi = \{i, \dots, n\}$  and  $F^\circ i = \{i+1, \dots, n\}$ , namely the upstream incremental distribution; and also including the well-known Shapley value, which is the barycenter of the core in convex games.

The cooperative core lower bounds are less easy to satisfy. If the river is shared among two or three agents, then there is always a distribution satisfying them. With four agents or more, the cooperative core lower bounds might not be satisfied in some river sharing problems. Ambec and Ehlers (2006) provide an example in which this is indeed the case with four agents. The logic is the following. If one of the two middle agents 2 and 3 is alone, it obtains its satiation benefit  $b_i(y_i)$  because the remaining agents pass some water through its location. As a consequence, agent 2's and 3's payoff should be higher than  $v(\{2\}) = b_2(y_2)$  and  $v(\{3\}) = b_3(y_3)$ , respectively. Moreover, each of the agents at the extremes of the river, 1 and 4, should get at least their stand-alone benefits,  $v(\{1\}) = b_1(e_1)$  and  $v(\{4\}) = b_4(e_4)$ , respectively. However, the total benefit of the efficient water allocation  $b_1(x_1^*) + b_2(x_2^*) + b_3(x_3^*) + b_4(x_4^*) = v(\{1, 2, 3, 4\})$  is strictly lower than

$b_1(e_1) + b_2(y_2) + b_3(y_3) + b_4(e_4)$ . Therefore, here it is impossible to distribute the benefit from  $x^*$  while giving every agent more than its stand-alone cooperative core lower bound  $v(\{i\})$ .<sup>7</sup>

To sum up, the set of transfers that are acceptable in the sense of the core (in that the members of any group obtain at least what they would get on their own) depends on the cooperative behavior of members outside the group. If they cooperate then this set might be empty, meaning that no (budget-balanced) transfer is acceptable. As a consequence, the agents might fail to implement the efficient allocation of water  $x^*$ . If they do not cooperate, then existence is guaranteed, and the set of transfers might be quite large. The next section reviews some fairness principles that may be used to select transfer schemes in this set.

## 6 Equity

While efficiency is defined by the application of the Pareto principle, there are many ways to define fairness or equity, depending on how people determine judgments that can be translated formally into axioms. The section starts by defining three axioms inspired by different judgments on what is fair. It then posits transfer schemes that satisfy the defined axioms while implementing the efficient allocation of water  $x^*$ . Yet some fairness axioms or criteria might not be compatible, thereby implying that no transfers satisfy all of them. The first two axioms, equal sharing individual rationality and envy-free, are standard in fair division problems. The third, the aspiration upper bounds, is a solidarity axiom that particularly suits the river sharing problem.

Note that the noncooperative core (or the noncooperative core lower bounds) can be seen as a fairness principle by itself. Without well-defined property rights for water, an agent or group of agents may claim property rights on the water it controls. For instance, in international river disputes, the principle of absolute territorial sovereignty grants to a country the right to water originating in its territory. It is then fair that the agent or group of agents obtains at least the benefit from consuming the water that it claims to own (Ambec and Sprumont 2002).

The first fairness principle is equal sharing individual rationality. It is based on the oldest axiom of the literature, on fair division, often taken as the definition of fairness (Steinhaus 1948; Moulin 1991). It stipulates that any agent should get at least the benefit of an equal division of the resources. Like the core lower bounds, it thus defines a lower bound on agents' payoffs.

The equal sharing individual rationality axiom can be easily adapted to the river sharing problem when all water originates from one source, that is, if  $0 = e_2 = e_3 = \dots = e_n$  and, therefore,  $e_1 = E_n$ , for instance when agents are farmers located along a canal devoted to irrigation and linked to a single water pool. In this case, an equal sharing of water means that everybody is entitled to claim at least  $\frac{e_1}{n}$ . In term of benefits or payoffs, it means that every agent  $i \in N$  should obtain at

least  $b_i\left(\min\left\{\frac{e_1}{n}, y_1\right\}\right)$ , assuming free disposal. The transfer scheme  $\mathbf{t}$  is thus equal sharing individual rational if:

$$b_i(x_i^*) + t_i \geq b_i\left(\min\left\{\frac{e_1}{n}, y_1\right\}\right) \text{ for every agent } i \in N. \quad (6.1)$$

With more than one tributary, one way to adapt the axiom is to assume that any agent  $i$  can claim an equal share of all tributaries located upstream (including  $e_i$ ). Formally, agent  $i$  has the right to a consumption level  $C_i = \frac{e_1}{n} + \frac{e_2}{n-1} + \dots + \frac{e_i}{n+1-i}$ . The transfer scheme  $\mathbf{t}$  is equal sharing individual rational if  $b_i(x_i^*) + t_i \geq b_i(\min\{C_i, y_i\})$  for every agent  $i \in N$ .

A second fairness principle also central to the axiomatic theory of justice is no envy (or envy-freeness), also called superfairness (Baumol 1986). According to the standard definition, an agent does not envy another agent if its payoff is higher with its assigned consumption bundle (here water and money) than it would be with the other agent's bundle. An allocation satisfies no envy if no agent envies the bundle assigned to another agent (Varian 1974).

The no envy principle can easily be defined in the river sharing problem when, again, all water inflow comes from one point, that is,  $0=e_2=e_3=\dots=e_n$ . In this case, all agents can claim to consume the water level assigned elsewhere in the river to another agent. Under free disposal, an allocation  $(\mathbf{x}^*, \mathbf{t})$  satisfies no envy (or is superfair) if

$$b_i(x_i^*) + t_i \geq b_i\left(\min\{x_j^*, y_i\}\right) + t_j \text{ for every } i, j \in N. \quad (6.2)$$

Similarly, a transfer scheme  $\mathbf{t}$  implements  $\mathbf{x}^*$  without envy if condition (6.2) is satisfied.

With tributaries ( $e_h > 0$  for  $h > 1$ ), the problem is that an agent located upstream might not be able to consume the level of water assigned to a downstream agent it envies because of lack of water at its location. One way to deal with this problem is to restrain envy-freeness to feasible water allocations; or, more precisely, to consider the feasible level of water the closest to the one the agents might envy. Formally, to consider  $E_i = e_1 + \dots + e_i$  as the alternative allocation for  $i$  if the bundle  $(x_j, t_j)$  it might envy is such that  $x_j < E_i$ . So, in the general case, an allocation  $(\mathbf{x}^*, \mathbf{t})$  satisfies feasible no envy (or a transfer scheme  $\mathbf{t}$  implements  $\mathbf{x}^*$  without feasible envy) if

$$b_i(x_i^*) + t_i \geq b_i\left(\min\{x_j^*, y_i, E_i\}\right) + t_j \text{ for every } i, j \in N. \quad (6.3)$$

Obviously if condition (6.2) holds, then so does condition (6.3). Therefore, if an allocation (or a transfer scheme) satisfies no envy according to the original definition, it also satisfies feasible no envy.<sup>8</sup>

Concerning the two above fairness axioms, Ambec (2006) provides a general result in the case of only one source  $e_1$ , assuming that the benefit functions are single crossing. Denote  $\lambda$  the shadow value of water with the efficient allocation, which is also the marginal benefit of agents at  $\mathbf{x}^*$ :

$$\frac{\partial b_i(x_i^*)}{\partial x} = \lambda \text{ for every } i \in N. \quad (6.4)$$

The transfers  $t_i^e = \lambda \left( \frac{e_1}{n} - x_i^* \right)$  for  $i=1, \dots, n$  implement the efficient allocation of water  $\mathbf{x}^*$  while satisfying both equal sharing individual rationality and no envy. Furthermore, when the number of agents is large and agents are sufficiently heterogeneous, these transfers are unique.

One way to achieve the transfer scheme  $\mathbf{t}^e$  is to price water or tax extraction at  $\lambda$  and to distribute the money collected equally. Then every agent  $i$  will extract water up to an equalized marginal benefit  $\lambda$ , thus selecting  $x_i^*$ . The total amount of money collected is thus  $\lambda(x_1^* + \dots + x_n^*) = \lambda e_1$ , where the last equality is due to the binding resource constraint. Each agent obtains a share  $\frac{e_1}{n}$  of

the money collected  $e_1$ . Therefore, each agent  $i$  obtains in the end  $b_i(x_i^*) - \lambda x_i^* + \lambda \frac{e_1}{n} = b_i(x_i^*) + t_i^e$ .

It will be clear later that another way to allocate money as in  $\mathbf{t}^e$  is to define property rights for water in a competitive water market.

Although the result in Ambec 2006 relies on the one water source case, it can be adapted to the general river sharing problem as follows. Consider the subsets  $N_1, \dots, N_K$  of  $N$  defined by the efficient allocation of water (see section 6.4). Denote  $\lambda_k$  the shadow value of water in the subset  $k$  for  $k=1, \dots, K$ . It is equal to the marginal benefit of agents in  $N_k$  and decreases strictly moving downstream from  $N_k$  to  $N_{k+1}$ . Denote also  $E(N_k) = e_i + \dots + e_j$  for any  $N_k = \{i, \dots, j\}$  for  $k=1, \dots, K$  the total flow of water controlled by members of  $N_k$ . Notice that efficiency requires that the agents in  $N_k$  share  $E(N_k)$  for  $k=1, \dots, K$ . Thus in a river sharing problem with one source  $E(N_k)$  shared by the

agents in  $N_k$ , following Ambec (2006),  $t_i^f = \lambda \left( \frac{E(N_k)}{|N_k|} - x_i^* \right)$  for every  $i \in N_k$  (where  $|N_k|$  denotes

the number of agents in  $N_k$ ) satisfies equal sharing individual rationality and no envy in the subset  $N_k$  for  $k=1, \dots, K$ . Since the no envy conditions are more stringent than the feasible no envy ones,  $\mathbf{t}^f = (t_1^f, \dots, t_n^f)$  defined above satisfies equal sharing (of  $E(N_k)$ ) individual rationality and feasible no envy among agents in  $N_k$  for  $k=1, \dots, K$ .

The third fairness principle is a solidarity axiom. It relies on the idea that, since water is scarce, everybody should make an effort. It starts by considering the welfare that an agent could achieve if it were alone on the river. In the absence of others, an agent  $i$  could consume up to the full

water stream originating from upstream of its location  $E_i = e_j + \dots + e_i$ . Call the benefit from consuming up to  $E_i$  the agent's aspiration welfare. Formally,  $i$ 's aspiration welfare is  $b_i(\min\{E_i, y_i\})$ . Of course, it is not possible to assign to every agent its aspiration welfare because the sum of the individuals' aspiration welfares exceeds the total welfare, that is,  $\sum_{i \in N} b_i(\min\{E_i, y_i\}) > \sum_{i \in N} b_i(x_i^*)$ . Therefore, by solidarity, no agent should end up with a welfare or payoff higher than its aspiration welfare, that is,  $b_i(x_i^*) + t_i \leq b_i(\min\{E_i, y_i\})$  for every  $i \in N$ . In Moulin's (1991) terms, since the river sharing problem exhibits negative group externalities, it is natural to ask that everyone takes up a share of these externalities. In addition, as argued in Ambec and Sprumont 2002, the aspiration upper bounds can be seen as an interpretation of the unlimited territorial integrity principle often invoked in international river conflicts (see Godona 1985; Sadoff, Whittington, and Grey 2003).

The above argument applies not only for individuals but also for coalitions. The aspiration welfare of a coalition  $S \subset N$  is the highest welfare it could achieve in the absence of MS. It is denoted by  $w(S)$  and formally defined in Ambec and Sprumont 2002. The aspiration welfare of a coalition  $S$  is the total benefit achieved by the members of coalition  $S$  if they share efficiently the full stream of water in the river. A transfer scheme that implements  $x^*$  satisfies the aspiration welfare upper bounds if any coalition welfare is lower than its aspiration welfare, formally,

$$\sum_{i \in S} b_i(x_i^*) + t_i \leq w(S) \text{ for every } S \subset N. \quad (6.5)$$

Ambec and Ehlers (2006) show that the unique transfer scheme that satisfies both the noncooperative core lower bounds and the aspiration welfare upper bounds is  $t^d$ : the one that implements the downstream incremental distribution. It yields to every agent  $i$  its marginal contribution to its predecessors, that is,  $b_i(x_i^*) + t_i^d = v(Pi) - v(P \circ i)$  for every  $i \in N$ . The next section addresses the issue of implementing the downstream welfare distribution with negotiation rules.

## 7 Implementation with negotiation rules

In practice, agents often negotiate to decide how much water each of them is entitled to consume. They may also bargain over compensation, as in the case of the Columbia River (Barrett 1994) or the Syr Darya River (Abbink, Moller, and O'Hara 2005).<sup>9</sup>

To coordinate international river management, countries often join institutions or sign treaties with specific negotiation rules. For instance, the "principe d'approbation des Etats" (principle of approval by the States) included in the treaty founding the Organisation pour la Mise en Valeur du Fleuve Sénégal (OMVS), an international institution that manages the Senegal River, forbids any member from changing the water flow without the consent of all others. Another example of

negotiation rules is the process leading to interstate river (or water) compacts to solve interstate river conflicts in the United States. These agreements are subject to congressional consent. In the case of disagreement, an allocation can be forced by the U.S. Supreme Court (Bennett and Howe 1998; Bennett, Howe, and Shope 2000). International treaties or negotiation rules sometimes come close to explicit game forms. This section describes a game, proposed in Ambec and Sprumont 2000, that implements the downstream incremental distribution as a subgame perfect equilibrium of this game.<sup>10</sup>

The game gives priority lexicographically to the most downstream user  $n, n-1, \dots, 2, 1$ . At the first stage, agent  $n$  proposes an allocation of water and money  $(\mathbf{x}, \mathbf{t})$  to the other agents in the river. Of course  $(\mathbf{x}, \mathbf{t})$  should be feasible:  $\mathbf{x}$  must satisfy the resource constraints at every location in the river and  $\mathbf{t}$  must be budget balanced. If all accept, the allocation is enforced. If at least one refuses, agent  $n$  leaves the negotiation table, obtaining the bundle  $(x_n, t_n) = (e_n, 0)$ . Then the next upstream agent  $n-1$  proposes a (feasible) allocation of water  $(x_1, \dots, x_{n-1})$  and money  $(t_1, \dots, t_{n-1})$  for the river sharing problem upstream. It is enforced if unanimously accepted. Otherwise, agent  $n-1$  gets  $(e_{n-1}, 0)$  and leaves the negotiation table. And the game proceeds this way until the last stage (if reached), in which agent 2 proposes a feasible water allocation  $(x_1, x_2)$  and budget-balanced transfer scheme  $(t_1, t_2)$  to 1, who accepts or refuses. It is enforced if 1 agrees. Otherwise, 2 gets  $(e_2, 0)$  and 1 gets  $(e_1, 0)$ . Straightforward backward induction shows that every subgame perfect equilibrium of this game implements the efficient allocation  $\mathbf{x}^*$  and the transfer scheme  $\mathbf{t}^d$  that yields the incremental welfare distribution.

## 8 Decentralization in water markets

For centuries markets for water have existed worldwide in irrigation communities (see Ostrom 1990 for case studies). Application of a market system is often recommended by economists to achieve efficiency because the inefficiency of free-access extraction is due to the lack of well-defined property rights for water. It thus seems natural to define property rights, leading to an efficient allocation of water on the premise that traders are price takers. But which rights? How should water be divided? Obviously, the assignment of property rights affects the payoffs of agents in the market through an allocation of money, leading to a transfer scheme in the river sharing problem. As for transfers, an allocation of property rights is acceptable by agents if it is perceived as fair.

In the case of a one-source river ( $e_2 = \dots = e_n = 0$ ), it is easy to show that equal division (of the water  $e_1$ ) leads to the equal sharing individual rational and envy-free transfer scheme  $\mathbf{t}^e$ . At the (competitive) market equilibrium, the agent's marginal benefits are all equal to the equilibrium price, which is then the shadow value of water  $\lambda$ . This equilibrium therefore implements  $\mathbf{x}^*$ . At

this price, any agent  $i$  buys or sells the difference between its endowment  $\frac{e_1}{n}$  and its efficient water consumption  $x_i^*$ . Agent  $i$  thus obtains  $\lambda \left( \frac{e_1}{n} - x_i^* \right) = t_i^e$ .

More generally, in any river sharing problem, equally dividing the water controlled by the agents in the subsets  $N_k$  of  $N$  (described in section 6.4) for  $k=1, \dots, K$  leads to a transfer scheme that satisfies no envy and equal sharing individual rationality among the members of  $N_k$  for  $k=1, \dots, K$ .

Notice that, in the one-source river sharing problem, equally splitting water might violate aspiration welfare upper bounds. Indeed, Ambec (2006) shows that the three fairness axioms outlined in section 6.6 might not be compatible, and posits a transfer scheme that implements  $\mathbf{x}^*$  while satisfying no envy, the aspiration welfare upper bounds, and the weaker requirement of individual rationality (agents' payoffs are nonnegative) in a one-source river sharing problem. The scheme can be implemented by pricing water or taxing extraction at  $\lambda$  but without redistributing the money collected.

## 9 Conclusion and policy implications

This synthetic review of the river sharing problem is now concluded by proposing some insights for public policies.

First, the analysis of the cooperative game helps to assess the potential gains from cooperation in the management of international or interstate rivers. There is no doubt that some form of transfer from downstream countries to upstream ones is needed to achieve efficiency. Such a transfer may take several forms, including of course direct monetary compensation, through a water market or fiscal transfers among states in a federal State, or compensatory payments through international treaties. But it could also take the form of a sharing rule of joint costs and benefits of utilities such as dams, canals, or hydropower plants, as for the Columbia River (Barrett 1994) and the Senegal River. Water can also be traded in exchange for other commodities, for example fuel supply on the Syr Darya River (Abbink, Moller, and O'Hara 2005) or electricity supply on the Mekong River between Thailand and Laos.

The likelihood of reaching an international river sharing agreement depends on the country's expectations about the status quo in the case of disagreement. If a country expects that the others will cooperate by reaching an agreement among them, it might be tempted to free ride on the agreement. Otherwise, an agreement is feasible. Nevertheless, for rivers shared by two or three countries, which is the case for many of them, an agreement is possible for any expectations. With more than three countries, it might not be manageable. These results might therefore provide some support for partial agreements with only the few main countries (for example Thailand and Laos on the Mekong).

To illustrate the main argument of the paper, let us consider as an example the section of the Nile River shared by Egypt, Sudan, and Ethiopia. The largest consumer, Egypt, is located downstream, whereas most of the flow (80%) originates from the most upstream country, Ethiopia. Egypt might be tempted to deal with Ethiopia to secure the water inflow exiting Ethiopian territory in exchange for some compensation. But then Sudan can free ride on the deal by extracting this increased supply of water entering in own territory without paying its cost (in the form of a compensation to Ethiopia). An inclusive agreement among the three countries must take into account this temptation to free ride by Sudan. With three countries or less, it is always possible to find such an inclusive agreement. But if one more country, for example Uganda, is included, no agreement might be acceptable by all. In this case, a partial agreement, for example between Egypt, Sudan, and Ethiopia, might be recommended.

Second, there are at least three reasons to expect or recommend the implementation of the downstream incremental distribution (or the transfer scheme  $t^d$ ) in international river agreements: (a) it assigns to every coalition of sovereign countries at least its noncooperative value (that is the welfare that this coalition can achieve by its own); (b) it is a compromise between two conflicting fairness principles invoked during international river disputes, the absolute territorial sovereignty and the unlimited territorial integrity; (c) it is the outcome of a game defined by simple negotiation rules. Basically, those rules assign more negotiation power to downstream countries than to upstream countries. If they adhere to the absolute territorial sovereignty and unlimited territorial integrity principles, countries might include these negotiation rules in international river sharing agreements.

In practice, river sharing treaties include international negotiation rules. For instance, the Indus Water Treaty establishes that a permanent Indus Commission is required to meet regularly to discuss potential disputes and to plan cooperative arrangements for the development of the basin. In the case of disagreement, the matter may be taken up by intergovernmental negotiations or, failing these, arbitration (Barrett 1994). In the United States, states must follow specific negotiation procedures to sign an interstate river water compact. The disagreement outcome is a solution imposed by the federal government (Bennett and Howe 1998).

Third, our axiomatic analysis of equitable transfer schemes might shed light on how to divide water among farmers producing irrigated crops. The analysis deals with heterogeneous farmers (differing land size, crops) sharing the same pool of water. The only way to sustain no envy and equal sharing individual rationality is to split this pool equally, providing that the water market is competitive and production functions satisfy some regularity property. If not, or if farmers are reluctant to market, one way to implement an efficient allocation of water complying with no envy and equal sharing individual rationality is to price or tax water at its shadow cost (which nevertheless has to be estimated) and to redistribute equally the money collected.

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## Endnotes

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<sup>1</sup> Complementary to this approach is that of Tsur and Dinar (1995), who examine the equity properties of real-world pricing methods for irrigation water.

<sup>2</sup> For a model of hydropower production with several agents, see Ambec and Doucet 2002.

<sup>3</sup> The notation  $N \setminus S$  refers to the set of agents in  $N$  outside  $S$ .

<sup>4</sup> A coalition is connected or consecutive if for all  $i, j \in S$  and all  $k \in N$ ,  $i < k < j$  implies  $k \in S$ .

<sup>5</sup> The analysis of noncooperative games between coalitions goes back to Aumann and Dreze 1974 and leads to the literature on stable coalition structures (for example Bloch 1996; Ray and Vohra 1997).

<sup>6</sup> This feature is common to cooperative games with externalities, such as the international pollution reduction game (Tulkens 1997).

<sup>7</sup> For extensions of the Shapley value for a cooperative game with externalities, see Maskin 2003 and Macho-Stadler, Pérez-Castrillo, and Wettstein, forthcoming.

<sup>8</sup> Sadoff, Whittington, and Grey (2003) also mention no envy or superfairness as a fairness principle that can be applied to the river sharing problem.

<sup>9</sup> See Carraro, Marchiori, and Sgobbi 2005 for a review of negotiation on water issues.

<sup>10</sup> See Moore 1992 for an introduction to the theory of implementation through game forms in complete information environments.